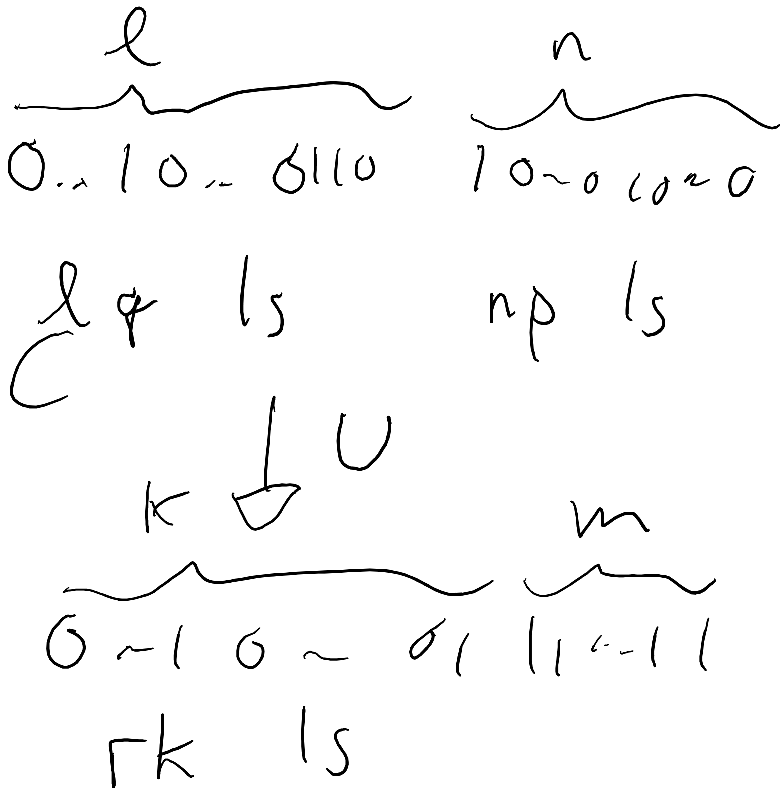


Constraints:

1) $k = l + n - m$: conserv. dim

2) $lq + np = rk + m$: conserv. energy

3) $2^{k h(r)} = 2^{l h(q) + n h(p)}$: conservation entropy (U is unitary)

$\varepsilon \equiv \frac{n}{l}, \; R \equiv \frac{m}{n}$

$$h(q) + \varepsilon h(p) = (1+\varepsilon - R\varepsilon)\, h\left(\underbrace{\frac{q+\varepsilon p - \varepsilon R}{1+\varepsilon - \varepsilon R}}\right)$$

$\varepsilon \to 0$ 1st ε

$$R = \frac{h(q) - h(p) + (p-q)/T}{h(q) + (1-q)/T}$$

$$= \frac{D(\rho \| \tau_H)}{D(|1\rangle\langle 1| \,\|\, \tau_H)} \quad \square$$

- work cost: similar argument
- ρ is diagonal energy basis

General case?

$$\rho^{\otimes n} \longmapsto \sum_E P_E \rho^{\otimes n} P_E \;\sigma$$

$\uparrow$ n possible energies

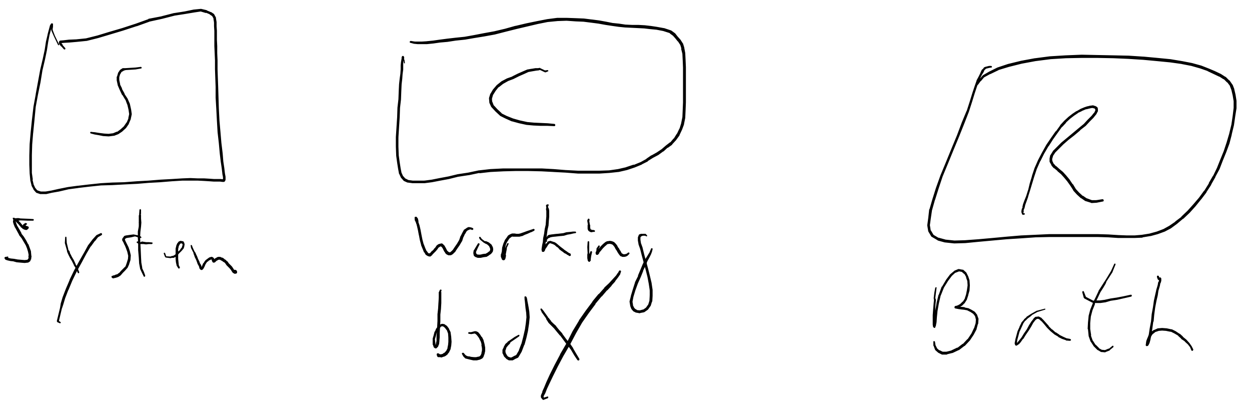
decohere by $\log(n)$ bits of entropy

$$S\left(\sum_E P_E \rho^{\otimes n} P_E \,\|\, \tau_H^{\otimes n}\right)$$

$$\approx n S(\rho \| \tau_H) + \underline{\log(n)}$$

work cost: non-diagonal need extra superposition over energy states $(O(\underline{\log(n)}))$

Calatytic Single copy transformations



$$\rho \xrightarrow[Cat]{TO} \sigma \quad \text{if } \exists\ \pi$$

$$\rho\otimes\pi \xrightarrow{TO} \sigma\otimes\pi$$

If ρ, σ are diagonal energy eigenbasis, $\rho \xrightarrow[Cat]{TO} \sigma$ —

iff $S_\alpha(\rho\|\tau_{H_S}) \geq S_\alpha(\sigma\|\tau_{H_S})$ $\forall$

$\alpha > 0$

$\alpha \geq 0$ ↑

Second laws

In general : only if

more constraints

Open question : what are all the constraints?

Open question : understand approximate catalysis

Open question : add adiabatic processes